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Patterns, Forms, Surfaces, Structures, Geometry, Origami

Geometry in Architectural Education

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ABSTRACT

Knowledge of geometry is essential for architectural education. Geometry is used for representing and communicating the design ideas. It is also used for design and development of architectural forms. The emphasis of education is on Descriptive Geometry for graphical representation of simple three-dimensional objects in two-dimensional forms. This knowledge is further used for design of buildings and preparing architectural drawings. The process is easy if the building forms are regular and they are associated with known concepts of geometry. With the increased use of computers, there is a tendency to design curved and irregular forms. It is necessary to study 'geometry of curved forms' and 'geometry of surfaces' to choose the appropriate structural form. Knowledge of geometry is essential for parametric design and digital processing.

This paper describes how geometry can contribute to the process of design and construction. The paper includes a discussion on various concepts and projects to create awareness about new challenges of digital design and structures

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Introduction

Knowledge of geometry is essential for architectural education. Geometry is used for representing and communicating the design ideas. It is also used for design and development of architectural forms. The emphasis of teaching is on Descriptive Geometry for representation of three-dimensional objects in two-dimensional forms. The studio exercise commonly comprises of drawing and drafting. It is observed that students complete the drawings mechanically, without much interest and enthusiasm. There are attempts by educationists to find out whether descriptive geometry is necessary, and also to use various methods to make the learning process interesting. In addition to descriptive geometry, concepts of 'symmetry-balance-composition', and concepts of 'proportion- Fibonacci numbers-golden ratio' are normally covered as a part of geometry education. Activity of 'documentation and measured drawings of buildings' is also common for students of Architecture. Most of the times, the emphasis of documentation is on visual representation of buildings. Architectural styles are compared for visual forms.

> **A review of works of Pier Luigi Nervi, Felix Candela, Fri Otto, Antonio Gaudi, and Buckminster Fuller shows that they have used concepts of geometry to experiment with structures and materials in their path breaking buildings. It is seen that curved surfaces and free-form structures designed by Architects like Frank Gehry, Foster + partners, and Zaha Hadid are based on geometric concepts in digital design processing.**

Works of great architects are appreciated for visual and spatial experiences. The significance of a geometrical form with respect to structure and method of construction is not given enough importance. A review of works of Pier Luigi Nervi, Felix Candela, Fri Otto, Antonio Gaudi, and Buckminster Fuller shows that they have used concepts of geometry to experiment with structures and materials in their path breaking buildings. It is seen that curved surfaces and free-form structures designed by Architects like Frank Gehry, Foster + partners, and Zaha Hadid are based on geometric concepts in digital design processing. It is quite clear that for analysis or for design of complex buildings, understanding of geometric properties of various structural forms is necessary. When the geometric concepts are clear, variations as response to the changing parameters are possible with the help of software tools. Knowledge about geometry of structures is also useful for making decisions about materials and methods of construction.

This paper is an attempt to explore potential of geometry as an allied subject to architectural design, construction, and structural design. Various projects and methods of learning are also discussed. This paper is based on workshops and projects conducted by the author as a part of ongoing research in the field of "Origami-Geometry- Architecture'. Some essential geometric terms related to shapes, forms, surfaces, and curvature are explained in detail along with classroom projects.

Use of Geometry in Architectural Education

Knowledge of geometric concepts as part of architectural education is expected to increase

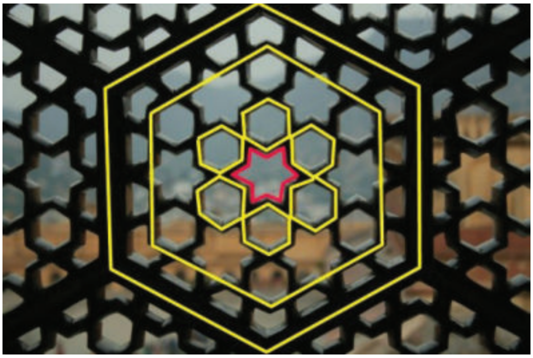

Figure 1: Hexagons and six pointed stars in a Jali

Figure 2: Hexagons and triangles in tiling

the potential of students in terms of the ability to conceive appropriate architectural form and the ability to realize the desired form in terms of structural system, materials, and methods of construction.

Objectives of learning geometry as part of Architectural education can be summarized as the following.

- To understand basic geometric forms
- To understand the geometry of existing
- architectural forms: visual analysis (learning activity)
- To explain structural aspects of form associated with geometric form: analysis of structures (teaching activity)

Here, I describe an academic experiment. The process of learning starts with observation of built forms, followed by experimentation and model making by hands. Once the basic concepts are clear, structural behavior and method of construction associated with geometric forms are discussed for some of the following structures.

- Types of sloping roofs associated with spans
- Types of masonry vaults and domes
- Long span structures and roofs for covering large areas in timber or steel
- Steel structures- domes (bucky balls, geodesic domes), spaces frames
- Form resistant concrete structures- domes, thin shells, and folded plate structures.
- Tensile fabric structures: Saddle roof, Mast supported, tent like forms, or conic forms
- Pneumatic structures
- Reciprocal Frame structures
- Tensegrity structures
- Kinetic structures (movable, deployable, convertible built forms)
- Grid shell structures in timber and steel
- Free-form structures

Essential Geometric Terms and Projects for Learning the Concepts

In this section, I describe some of the essential geometrical terms such as polygons, polyhedral, platonic solids, polyhedral surfaces, tessellated roofs etc. Further, a few simple student exercises are described that helped them to grasp these concepts.

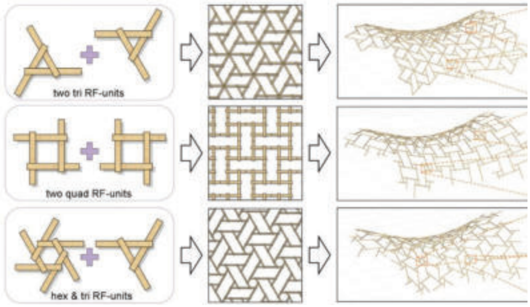

Figure 3: Tessellation to reciprocal frame structures (from Song, 2013.) ***Source:*** *staff.ustc.edu.cn/*

Polygons: Polygons are closed shapes with 'n' number of sides; a regular convex polygon is constructed by connecting equidistant points on a circle. Only three types of polygons- triangles, squares, and hexagons can be used to form regular tessellations (tiling) to cover infinite area. Polygons are used for tiling the floors or for covering large roof areas. It is also possible to use combinations of polygons, or irregular polygons. For a flat surface, at any intersection point the total of all the angles will be exactly 360 degrees.

Observation and Photo Documentation: Students were asked to observe polygons and other shapes in buildings and interior spaces. They were asked to take photographs of grills, jalis, paving, and tiling. They were asked to depict the shapes (**figure 1, 2**). This activity was taken further by some students to study the Islamic geometric patterns.

Use of Grid Paper: Students were asked to use graph papers and square ruled papers to understand the grid formation by parallel lines. They were asked to create square grid as lines or dots. It was very easy to understand the concept of scale by relating the length of a segment with the distance between the points. The concept of graphic scale was also explained. 45 degree set squares were used to draw diagonal lines. Triangle grids were formed by using 30-60 set squares. Patterns were drawn by using the lines or by connecting the dots. Some students could relate this exercise with traditional Indian *Rangoli* patterns with dots. Triangular-grid papers were also used to draw isometric views. It is observed that if the concepts of grid formation are clear, students can explore the patterns for diagrid structures, space frames, and grid shell structures.

Tiling and Tessellation: Tiling is the arrangement of shapes to cover the surface without any gaps or overlaps. Tiling is possible with polygons, modified polygons, or any other shapes. **Figure 2** shows tiling pattern with hexagons and triangles. Tessellation is another word for tiling. For tessellations as an art form, the 'tile' is any decorative motif or irregular shape. A tessellation is created when a shape is repeated again and again without any overlaps or gaps. Tessellations can be recognized by the number of different types of tiles involved, and by the regularity, symmetry, and periodicity of the pattern. The tile can repeat in many ways. There are three basic ways of transformation of a tile- Translation (slide or glide), Reflection (flip), and Rotation (turn). Depending upon the shapes, these methods can be combined. Another way of transformation is resizing (expansion or contraction) (Wilson, 1983).

Students used polygonal paper tiles to cover paper surface. The tessellation patterns were drawn on papers, and also by using computer software tools. Study of related literature shows that it is possible to use stick-modules to create units of

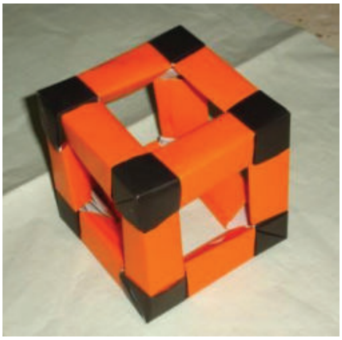

Figure 4 : Cube

Figure 5 : Dodecahedron

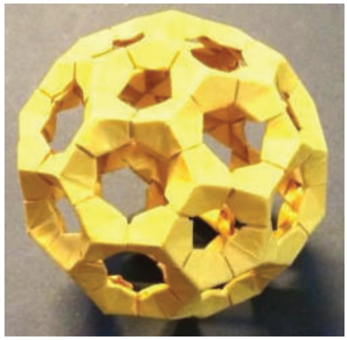

Figure 6 : Bucky Ball

reciprocal frames, and combined to form large patterns. These patterns are deformed using computer software tools to create reciprocal frame structures, as seen in **figure 3**.

Polyhedron: A polyhedron is a solid with polygonal faces, straight edges, and points as vertices. Students were asked to use polygons to make models. They used origami face modules or edge modules. They used three edge modules forming 90 degree angle for a cube corner (**Figure 4**). Three pentagons were used for a corner of dodecahedron (**Figure 5**). It was realized that they can create three-dimensional forms as long as the total of all angles at any vertex point is less than 360 degrees. It was obvious that they could create only five regular polyhedral forms. These forms are known as platonic solids.

Platonic Solids: A Platonic solid is a regular convex polyhedron. It is constructed by congruent regular polygonal faces with the same number of faces meeting at each vertex. The five platonic solids are Tetrahedron (4 triangles), Cube or Hexahedron (6 squares), Octahedron (8 triangles), Dodecahedron (12 pentagons), and Icosahedron (20 triangles).

Polyhedral Forms: Combinations of polygons are used to create various types of polyhedral forms. Some of these solids are observed in daily life. Traditional Diwali lantern is a combination of eight triangles and six squares. A football has twelve pentagons and twenty hexagons. **Figure 6** shows a model of Bucky Ball using edge modules for combination of hexagons and pentagons. The knowledge of polygons and polyhedral forms is further used to understand concepts of geodesic domes, tessellated roofs, reciprocal roofs, grid shells, and curved wall surfaces. Students were also asked to make products using polygons and polyhedral forms. The most popular products were lampshades and lanterns. **Figure 7** shows the use of reducing square frames arranged in a spiral pattern. Modular Icosahedron is seen in **figure 8**, and **figure 9** shows paper lantern in the form of a snub cube.

Study of Surfaces and Curvature

Students were asked to make models of buildings using flat sheets of paper. They were allowed to cut, fold, or roll the sheet. It was realized that it is possible to make models of flat or sloping roofs, pyramidal roofs, conical roofs, and even barrel vault roofs, but models for domes and shell roofs

Figure 7 : Reducing Square

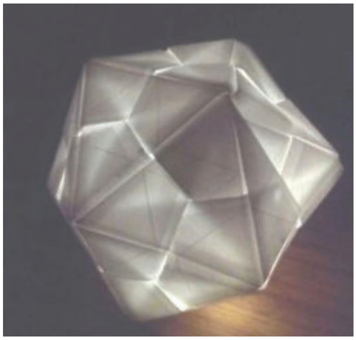

Figure 8 : Icosahedron

Figure 9 : Snub Cube

cannot be made by using a continuous sheet of paper. It was explained to them that the types of surfaces are different for different geometric forms of buildings.

Geometric Terms for Surfaces:

In this section, I describe a few important geometric terms pertaining to Surfaces such as- developable surface, rotational surface, translational surface, ruled surface as well as foldable, kinetic, and free form. Once again, student exercises are described which helped in understanding them.

Developable Surface: In mathematics, a developable surface[1] is a smooth surface with zero Gaussian curvature. That is, it is a surface that can be flattened onto a plane without distortion (i.e. 'stretching' or 'compressing'). Conversely, it is a surface which can be made by transforming a plane (i.e. 'folding', 'bending', 'rolling', 'cutting' and/or 'gluing'). **Figure 10** shows some examples of development of surfaces of polyhedral forms, and **figure 11** shows use of paper strips for a cylinder, prism and other forms. A circle is cut and glued to form a cone.

Rotational Surface: Rotational surfaces[2] are described by the rotation of a plane curved around a vertical axis. The curve can have variety of shapes, and can give rise to variety of dome forms. A surface described by rotating an inclined line around a vertical axis is a cone. Conical surfaces are called as umbrellas when supported at a point and as domes when supported on their circular boundary.

Translational Surface: Translational surface[3] is obtained by translating or sliding a plane curve on another plane curve or straight line. A cylindrical roof (barrel roof) is obtained by sliding a half round curve along a straight line.

Ruled Surface: Any surface generated by sliding the two ends of a line segment on two separate curves is called as a ruled surface.[4] A cylinder is a ruled surface formed by translation of a straight vertical segment on two horizontal circles lying one above the other. An inclined segment sliding on two horizontal circles lying one above the other is called as hyperboloid of one sheet. (**Figure 12**).

As ruled surfaces are formed by line segments, models for ruled surfaces were constructed by sticks or paper strips. A frame was prepared with two points at one level and diagonally opposite points at higher levels. When sticks were connected between these frames a doubly curved surface (hyperbolic paraboloid) was obtained as shown in **figure 13.** A square frame with central point at lower level was covered with paper strips as shown in **figure 14** to obtain an inverted umbrella roof form. It was much easier to explain the concepts of curvature with help of these models.

Geometric Terms for Curvature

Here I discuss, the terms pertaining to curvature such as curves, single curvature, double curvature, foldable or deployable surface etc. as also a few related exercises.

Curves: Plane curves (circle, ellipse, parabola, and hyperbola) are studied as sections of a cone. Students were asked to use graphical methods and digital methods to draw these curves. They were also encouraged to study mathematical equations of these curves.

Curvature: A curvature[5] of a surface at a point is exhibited by cutting it with a plane perpendicular to it. The curvature varies as the plane rotates. In case of a dome the curvature is downwards, in case of a dish the curvature is upward at all the points. Domes have positive curvature and dishes have negative curvatures. These surfaces are called as non-developable surfaces because they cannot be flattened without stretching them, unless they are cut at number of infinite sections.

A horse saddle has downwards curvature across the horse and upward curvature along the horse. If the saddle is cut by a vertical rotating plane, its curvature changes not only in value but also in sign. As the cutting plane rotates along its axis, the curvature changes gradually from positive to negative values and back from negative to positive values. Such surfaces are called as anticlastic surfaces. These surfaces are also non-developable (Salvadori M., 1963).

Making a Tensile Structure: As students had already studied double curvature of ruled surfaces, the concept of tensile structures was introduced by use of stretch fabric and wire frames. Students made models of tensile structures by stretching the fabric or by twisting the frame. (F**igures 15, 16. 17**).

Foldable surfaces or deployable surfaces: Deployable surfaces are surfaces that can fold, either for transportation or for storage. According to Mark Schenk (2012), surfaces can be flat-foldable when in a folded state the object lies in a plane. For a rigid foldable object, the folding process is more important rather than the final folded state. The material only bends at the fold lines.

Robert J. Lang (2016) developed deployable structures based on origami flasher. As shown in F**igure 18**, it is possible to wrap a square sheet as a ring.

Use of Origami: It was necessary to understand the Origami crease patterns (combinations of ridges and valleys) associated with different folding actions. Most useful patterns like 'Miura Ori' pattern for flat folding, 'Curved Miura' and 'Yoshimura' patterns were folded from flat paper sheets. Examples of deployable fabric forms as temporary shelters were presented to the students.

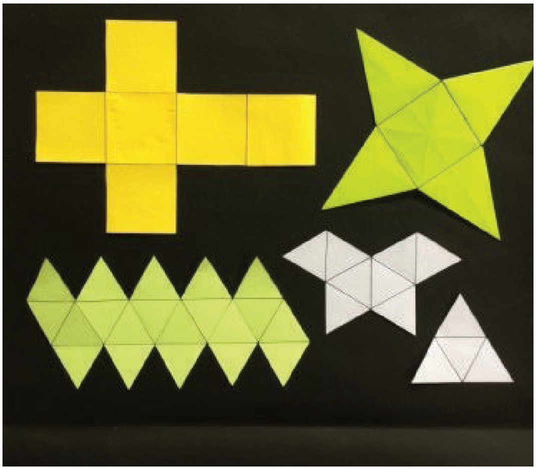

Figure 10: *Surface development of polyhedral forms*

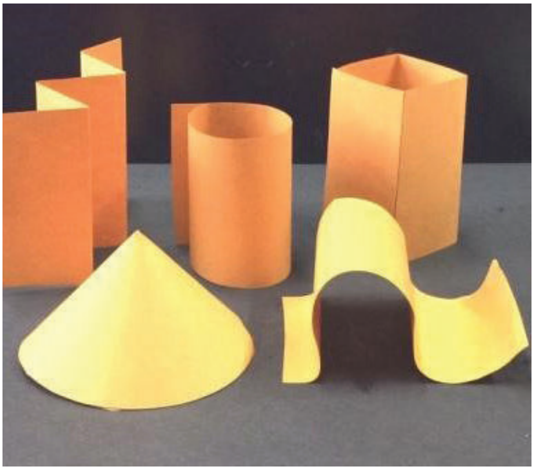

Figure 11: *Concept of developable surfaces*

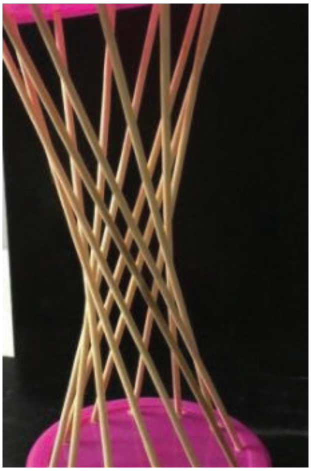

Figure 12: *Hyperboloid*

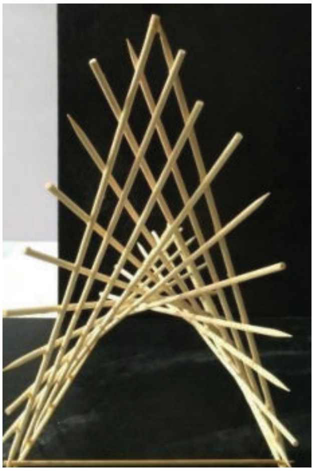

Figure 13: *Hyperbolic paraboloid*

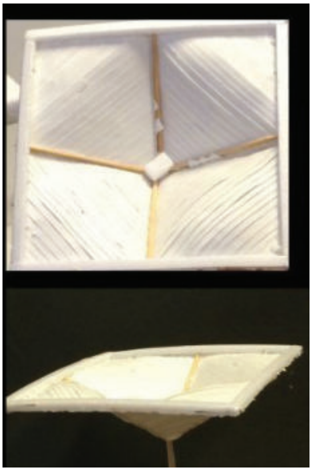

Figure 14: *Inverted umbrella roof*

Figure 15: *Doubly curved*

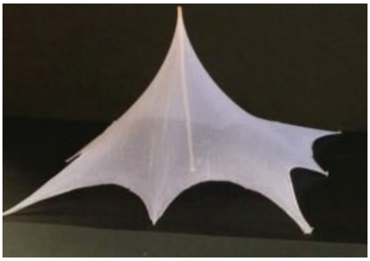

Figure 16: *Mast supported*

Figure 17: *Saddle*

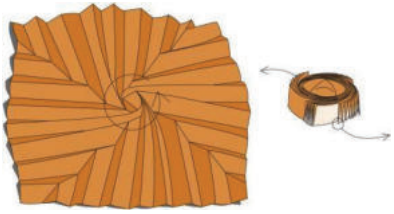

Figure 18: *Flasher (Wrapping)*
Source: *Credit: Robert Lang (2016)*

Figure 19: *Curved Miura*

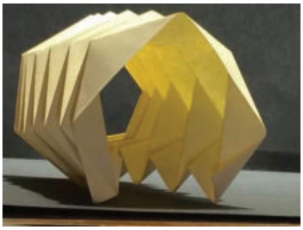

Figure 20: *Yoshimura cylinder*

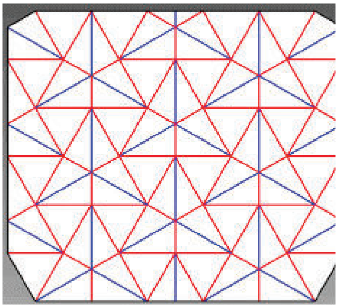

Figure 21: *Ron Resch pattern*
Source: *ronresch.org*

Figure 22: *Ceiling of resonant chamber.* ***Source:*** *archdaily.com*

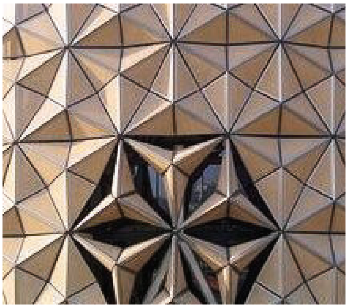

Figure 23: *Façade of Al-Bahar Towers, Abu Dhabi.* ***Source:*** *archdaily.com*

Kinetic Structures and Surfaces: Folding and Parametric Design

The mathematician and Designer Ron Resch was one of the first who explored the potential of three dimensional tessellated surfaces. He introduced the use of computers and software tools to experiment with origami folding units and tessellated units to develop overall forms (Cueva, 2013). Use of Ron Resch crease pattern (**Figure 21**) for a surface allows stimulation of triangle water bomb units to expand and contract. **Figure 22** shows ceiling of acoustic studio where triangular panels connected by hinges contract and expand to achieve desirable absorptive and reflective surface (Furuto, 2012). **Figure 23** shows triangular water bomb panels on the façade of Al- Bahar Towers in Abu Dhabi, which open and close to control the amount of sunlight entering the building.

> **With the current software tools it is possible to design free-form surfaces but fabrication of large-scale structures is a major challenge. One has to decompose the surfaces into smaller panels which can be manufactured, provide appropriate support structures, meet structural constraints, and make sure that the cost does not become excessive.**

Free Form Structures and Surfaces: Geometry and Parametric Design

A Free-Form surface[6] is a surface that cannot be created by sweeping a 2-dimensional curve around an axis or sweeping it along a straight line. According to Helmut Pottmann (2015), free forms constitute one of the major trends within contemporary architecture. With the current software tools it is possible to design free-form surfaces but fabrication of large-scale structures is a major challenge. One has to decompose the surfaces into smaller panels which can be manufactured, provide appropriate support structures, meet structural constraints, and make sure that the cost does not become excessive. Many of these problems are actually of geometric nature, and can be resolved by geometric modeling and geometric processing.

Some of the associated concepts are as follows.

- Polyhedral surface
- Developable panels and semi discrete models –skins from smooth strips
- Curved folding
- Use of smooth doubly curved skins
- Paneling
- Geometric support structures

Analysis of Geometric Forms in Building Projects

Students were encouraged to study the existing buildings and analyze them for wall surfaces, roof structures, and materials used. The teaching activity was continued in the technology studio where various geometric forms associated with structural forms, and materials were discussed. Some examples are stated here.

- The surface for a dome can be defined as a rotational surface; the form of the dome will be decided by the shape of the rotating curve. Possible materials are stone or brick masonry, concrete, or ferro-cement. In case of a geodesic dome, the surface is polyhedral; the frame work can be of timber, steel, or bamboo.
- Grid shell structures are constructed in steel or timber; they are structurally different from space frame structures. Some grid shell structures are visually similar to reciprocal frame structures, but the method of construction is different. Grid shell structures are constructed with lattice of continuous members that span from one end of the structure to the other end. This means that the structure is often laid out flat and lifted or otherwise deformed into its intended form. In case of reciprocal frames, short beams are used that terminate on the length of a neighboring member, and provide termination points to the other member (Danz, 2014).
- Folded plates show variations of geometrical forms as parallel, radial, or conical. The material used depends on the span and can be timber, concrete, or ferro-cement.
- Doubly curved structures with ruled surfaces can be constructed as concrete thin shells or tensile fabric roofs, or assembled from timber or bamboo.
- Curved wall surfaces are possible by using diagrid structural system, and a number of flat polygonal panels. This can be seen in the Swiss Re Headquarters in London, designed by Foster + partners, also known as Gherkin (Brady Peters, 2008).
- Some of the free-form structures by Frank Gehry show use of developable surfaces constructed from sheet metal, such as his Walt Disney Concert Hall in Los Angeles. It is possible to cover large open areas by using free-form triangle mesh or quadrilateral mesh. This can be seen in the Great Court of the British Museum, London by Foster + partners (Helmut Pottmann, 2015)

Discussion

Knowledge of Geometry is important for students of architecture. Geometric concepts are used to achieve satisfaction in terms of

> **The use of geometry in architectural education should not be limited to representation; it should be used for design and construction of complex forms. Knowledge of Geometry is useful for traditional and specialized methods of construction. It is equally important for digital processing and computation.**

visual and spatial aspects of built form. It is also necessary to study structural behavior of a form associated with geometric form, and suitability of the material for wall surfaces and roofing members. Students can make study models, and/or use computer software tools to study and use desired geometric forms. The use of geometry in architectural education should not be limited to representation; it should be used for design and construction of complex forms. Knowledge of Geometry is useful for traditional and specialized methods of construction. It is equally important for digital processing and computation. ■

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Notes:

[1] https://en.wikipedia.org/wiki/Developable_surface

[2] M. Salvadori and R. Heller (1963), Structures in Architecture, Prentice Hall, New Jersey, p. 328

[3] ibid, p. 332

[4] ibid, p. 336

[5] ibid, p. 324

[6] http://forum.zemax.com/4739/Definition-Of-Free-Form-Surfaces

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